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Natural R–Parity, μ–term, and Fermion Mass Hierarchy From Discrete Gauge Symmetries

K.S. Babu[1], Ilia Gogoladze[2] and Kai Wang[3]

Department of Physics, Oklahoma State University
Stillwater, OK 74078, USA

Abstract

In the minimal supersymmetric Standard Model with seesaw neutrino masses we show how R–parity can emerge naturally as a discrete gauge symmetry. The same discrete symmetry explains the smallness of the μ–term (the Higgsino mass parameter) via the Giudice–Masiero mechanism. The discrete gauge anomalies are cancelled by a discrete version of the Green–Schwarz mechanism. The simplest symmetry group is found to be Z_4 with a charge assignment that is compatible with grand unification. Several other Z_N gauge symmetries are found for $N = 10,\ 12,\ 18,\ 36$ etc, with some models employing discrete anomaly cancellation at higher Kac–Moody levels. Allowing for a flavor structure in Z_N, we show that the same gauge symmetry can also explain the observed hierarchy in the fermion masses and mixings.

[1]E-mail address: babu@okstate.edu
[2]On leave of absence from: Andronikashvili Institute of Physics, GAS, 380077 Tbilisi, Georgia. E-mail address: ilia@hep.phy.okstate.edu
[3]E-mail address: wk@okstate.edu

1 Introduction

One of the challenging questions facing supersymmetric extensions of the Standard Model is an understanding of R–parity which is required for the stability of the proton. In the minimal supersymmetric Standard Model (MSSM), a discrete Z_2 symmetry is usually assumed. Under this symmetry the Standard Model (SM) particles are taken to be even while their superpartners are odd. The gauge symmetry of MSSM would allow baryon number and lepton number violating Yukawa couplings at the renormalizable level which would result in rapid proton decay. The Z_2 R–parity forbids such dangerous couplings.

The assumption of R–parity has profound implications for supersymmetric particle search at colliders as well as for cosmology. At colliders SUSY particles can only be produced in pairs, and the lightest SUSY particle (LSP), usually a neutralino, will be stable. This stable LSP is a leading candidate for cosmological cold dark matter.

Since R–parity is not part of the MSSM gauge symmetry, questions can be raised about its potential violation arising from quantum gravitational effects. These effects (associated with worm holes, black holes, etc) are believed to violate all global symmetries [1]. True gauge symmetries are however protected from such violations. When a gauge symmetry breaks spontaneously, often a discrete subgroup is left intact. Such discrete symmetries, called discrete gauge symmetries [2], are also immune to quantum gravitational effects. Not all discrete symmetries can however be gauge symmetries. For instance, since the original continuous gauge symmetry was free from anomalies, its unbroken discrete subgroup should be free from discrete gauge anomalies [3, 4]. This imposes a non–trivial constraint on the surviving discrete symmetry and/or on the low energy particle content [2, 3, 4, 5, 6, 7, 8].

It will be of great interest to see if R–parity of MSSM can be realized as a discrete gauge symmetry, so that one can rest assured that it wont be subject to unknown quantum gravitational violations. This is the main question we wish to address in this paper.

A seemingly unrelated but equally profound problem facing the MSSM is an understanding of the origin of the μ–term, the Higgsino mass parameter. The μ parameter is defined through the superpotential term $W \supset \mu H_u H_d$, where H_u and H_d are the two Higgs doublet superfields of MSSM. Since the μ–term is SUSY–preserving and is a singlet of the SM gauge symmetry, its natural value would seem to be of order the Planck scale. But $\mu \sim 10^2$ GeV is required for consistent phenomenology. It will be desirable, and is often assumed, that the μ term is related to the supersymmetry breaking scale. An attractive scenario which achieves this is the Guidice–Masiero mechanism [9] wherein a bare μ term in the superpotential is forbidden by some symmetry. It is induced through a higher dimensional Lagrangian term

$$\mathcal{L} = \int d^4\theta \frac{H_u H_d Z^*}{M_{\rm Pl}} \tag{1}$$

where Z is a spurion field which parametrizes supersymmetry breaking via $\langle F_Z \rangle \neq 0$, with $\langle F_Z \rangle / M_{\rm Pl} \sim M_{\rm SUSY} \sim 10^2$ GeV. For this mechanism to work, there must exist a symmetry that forbids a bare μ term in the superpotential. Such a symmetry cannot be a continuous symmetry, consistent with the requirement of non–zero guaginos masses, and therefore must be discrete. It would be desirable to realize this as a discrete gauge symmetry.

The purpose of this paper is show that it is possible to realize Z_N symmetries as discrete gauge symmetries which act as R–parity and which solve simultaneously the μ–problem via the Guidice–Masiero mechanism. We make use of a discrete version of the Green–Schwarz mechanism [10] for anomaly cancellation. Simple realizations of R–parity as a discrete gauge symmetry are possible which also solve the μ–problem, without enlarging the particle content of MSSM. The simplest symmetry group we have found is a Z_4. Under this Z_4 all MSSM matter superfields and the gauginos carry equal charge of 1 while the Higgs superfields have zero charge. Such a simple charge assignment is compatible with grand unification. This charge assignment is anomaly–free by virtue of the discrete Green–Schwarz mechanism. Other Z_N symmetries with $N = 10,\ 12,\ 18,\ 36$, etc are also identified, some realized at higher Kac–Moody level. Either lepton parity or baryon parity can be obtained as a discrete symmetry in this approach, with baryon parity requiring anomaly cancellation at higher Kac–Moody levels. By allowing for a family–dependent structure in Z_N, we show how it is possible in our framework to explain the observed fermion mass and mixing hierarchy in a simple way.

Attempts have been made in the past to derive R–parity as a discrete gauge symmetry within MSSM. Early analyses [5, 6] did not incorporate the seesaw mechanism for neutrino mass or the Guidice–Masiero mechanism for generating the μ–term. A recent analysis which includes these features [7] has found Z_9 and Z_{18} discrete gauge symmetries as possible candidates for R–parity by demanding these symmetries to be anomaly–free. It turns out that in these models [7] the Kahler potential violates R–parity at higher order, leading to cosmologically disfavored lifetime for the neutralino LSP. Furthermore, the discrete charge assignment in these models was not compatible with grand unification with the MSSM spectrum. The main difference in our approach is that we make use of the Green–Schwarz mechanism for discrete anomaly cancellation, which is less restrictive compared to the straightforward methods. The outcome differs in several ways, notably in the realization of simpler symmetries (eg. Z_4), exact R–parity without cosmological problems, and compatibility with grand unification. It should be mentioned that enlarging the particle content of MSSM has been proposed as a solution to the R–parity and μ problems [8]. In contrast to such approaches, in our framework, the low energy spectrum is identical to that of MSSM.

This paper is organized as follows. In Sec. 2 we review briefly the discrete version of Green–Schwarz anomaly cancellation mechanism. Sec. 3 contains our main results. In 3.1 we write down the constraints arising from the Lagrangian of MSSM and the discrete anomaly cancellation conditions. In Sec. 3.2 we identify possible discrete gauge symmetries at Kac–Moody level 1 which prevent R–parity violating terms. In Sec. 3.3 we embed these symmetries to a higher Z_N to solve the μ–problem. Sec. 3.4 is devoted to solutions based on higher Kac–Moody levels. In Sec. 4 we show how a simple discrete gauge symmetry can explain the fermion mass and mixing angle hierarchy. Finally we conclude in Sec. 5.

2 Discrete anomaly cancellation via Green–Schwarz mechanism

Let us first recall the essence of the Green–Schwartz (GS) anomaly cancellation mechanism for a $U(1)$ guage symmetry. String theory when compactified to four dimensions generically contains an "anomalous $U(1)_A$" gauge symmetry. A subset of the gauge anomalies in the axial vector $U(1)_A$ current can be cancelled via the GS mechanism in the following way [10]. In four dimensions, the Lagrangian for the gauge boson kinetic energy contains the terms

$$\mathcal{L}_{\text{kinetic}} = \varphi(x) \sum_i k_1 F_i^2 + i\eta(x) \sum_i k_i F_i \tilde{F}_i \tag{2}$$

where $\varphi(x)$ denotes the string dilaton field and $\eta(x)$ is its axionic partner. The sum i runs over the different gauge groups in the model, including $U(1)_A$. k_i are the Kac–Moody levels for the different gauge groups, which must be positive integers for the non–Abelian groups, but may be non–integers for Abelian groups. The GS mechanism makes use of the transformation of the string axion field $\eta(x)$ under a $U(1)_A$ gauge variation $V_A^\mu \rightarrow V_A^\mu + \partial_\mu \theta(x)$,

$$\eta(x) \rightarrow \eta(x) - \theta(x)\delta_{GS} \tag{3}$$

where δ_{GS} is a constant. If the anomaly coefficients involving the $U(1)_A$ gauge boson and any other pair of gauge bosons are in the ratio

$$\frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3}, ... = \delta_{GS} \ , \tag{4}$$

these anomalies will be cancelled by gauge variations of the $U(1)_A$ field arising from the second term of Eq. (2). δ_{GS} in Eq. (4) is also equal to the mixed gravitational anomaly, $\delta_{GS} = A_{\text{gravity}}/12$ [11]. All other crossed anomaly coefficients should vanish, since they cannot be removed by the shift in the string axion field.

Consider the case when the gauge symmetry in four dimensions just below the string scale is $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_A$. Let A_3 and A_2 denote the anomalies associated with $[SU(3)_C]^2 \times U(1)_A$ and $[SU(2)_L]^2 \times U(1)_A$ respectively. Then if $A_3/k_3 = A_2/k_2 = \delta_{GS}$ is satisfied, from Eq. (4), it follows that these mixed anomalies will be cancelled. The anomaly in $[U(1)_Y^2] \times U(1)_A$ can also be cancelled in a similar way if $A_1/k_1 = \delta_{GS}$. However, in practice, this last condition is less useful, since k_1 is not constrained to be an integer as the overall normalization of the hypercharge is arbitrary. If the full high energy theory is specified, there can be constraints on A_1 as well. For example, if hypercharge is embedded into a simple group such as $SU(5)$ or $SO(10)$, $k_1 = 5/3$ is fixed since hypercharge is now quantized. $A_1/k_1 = \delta_{GS}$ will provide a useful constraint in this case. We shall remark on this possibility in our discussions. Note also that cross anomalies such as $[SU(3)] \times [U(1)_A]^2$ are automatically zero in the Standard Model, since the trace of $SU(N)$ generators is zero. Anomalies of the type $[U(1)_Y] \times [U(1)_A]^2$ also suffer from the same aribitrariness from the Abelian levels k_1 and k_A. Finally, $[U(1)_A]^3$ anomaly can be cancelled by the GS mechanism, or by contributions from fields that are complete singlets of the Standard Model gauge group.

The anomalous $U(1)_A$ symmetry is expected to be broken just below the string scale. This occurs when the Fayet–Iliopoulos term associated with the $U(1)_A$ symmetry is cancelled, so that supersymmetry remains unbroken near the string scale, by shifting the matter superfields that carry $U(1)_A$ charges [12]. Although the $U(1)_A$ symmetry is broken, a Z_N subgroup of $U(1)_A$ can remain intact. Suppose that we choose a normalization wherein the $U(1)_A$ charges of all fields are integers. (This can be done so long as all the charges are relatively rational numbers.) Suppose that the scalar field which acquires a vacuum expectation value (VEV) and breaks $U(1)_A$ symmetry has a charge N under $U(1)_A$ in this normalization. A Z_N subgroup is then left unbroken down to low energies. We shall identify R–parity of MSSM with this unbroken Z_N symmetry.

The field that acquires a VEV and breaks $U(1)_A$ to Z_N can supply large masses of order the string scale to a set of fermions which have Yukawa couplings involving this field. Such fields may include Majorana fermions and Dirac fermions. These heavy fields can carry SM gauge quantum numbers, but they must transform vectorially under the SM. In order that their mass terms be invariant under the unbroken Z_N, it must be that

$$\begin{aligned} 2q_i &= 0 \ mod\ N \text{ (Majorana fermion)} \\ q_i + \bar{q}_i &= 0 \ mod\ N \text{ (Dirac fermion)} \end{aligned} \tag{5}$$

where q_i are the $U(1)_A$ charges of these heavy fermions. The index i is a flavor index corresponding to different heavy fields. These heavy fermions, being chiral under the $U(1)_A$, contribute to gague anomalies. Their contribution to the $[SU(3)_C]^2 \times U(1)_A$ gauge anomaly is given by $A_3 = \sum_i q_i \ell_i = (N/2)\sum_i p_i \ell_i$ (Majorana fermion) or $A_3 = \sum_i (q_i + \bar{q}_i)\ell_i = (N)\sum_i p_i \ell_i$ (Dirac fermion) where ℓ_i is the quadratic index of the relevant fermion under $SU(3)_C$ and the p_i are integers. We shall adopt the usual normalization of $\ell = 1/2$ for fundamental of $SU(N)$. Then, for the case of heavy Dirac fermion, one has $A_3 = p(N/2)$ where p is an integer, as the index of the lowest dimensional (fundamental) representations is 1/2 and those of all other representations are integer multiples of 1/2. The same conclusion follows for the case of Majorana fermions for a slightly different reason. All real representations of $SU(3)_C$ (such as an octet) have integer values of ℓ, so that $\sum_i p_i \ell_i$ is an integer. Analogous conclusions follow for the $[SU(2)_L]^2 \times U(1)_A$ anomaly coefficient.

If the Z_N symmetry that survives to low energies was part of $U(1)_A$, the Z_N charges of the fermions in the low energy theory must satisfy a non–trivial condition: The anomaly coefficients A_i for the full theory is given by A_i from the low energy sector plus an integer multiple of $N/2$. These anomalies should obey Eq. (4), leading to the discrete version of the Green–Schwarz anomaly cancellation mechanism:

$$\frac{A_3 + \frac{p_1 N}{2}}{k_3} = \frac{A_2 + \frac{p_2 N}{2}}{k_2} = \delta_{GS} \tag{6}$$

with $p_1,\ p_2$ being integers. Since δ_{GS} is an unknown constant (from the effective low energy point of view), the discrete anomaly cancellation conditions of Eq. (6) are less stringent than those arising from conventional anomaly cancellations. If $\delta_{GS} = 0$ in Eq. (6), the anomaly is cancelled without assistance from the Green–Schwarz mechanism. We shall not explicitly use the condition that $\delta_{GS} \neq 0$, so our solutions will contain those obtained by demanding $\delta_{GS} = 0$ in Eq. (6), viz., $A_3 = -p_1(N/2)$, $A_2 = -p_2(N/2)$ with $p_1,\ p_2$ being integers.

In our analysis we shall not explicitly make use of the condition $A_1/k_1 = A_2/k_2$, since, as mentioned earlier, the overall normalization of hyperhcarge is arbitrary. However, once a solution to the various Z_N charges is obtained, we can check for the allowed values k_1, and in particular, if $k_1 = 5/3$ is part of the allowed solutions. This will be an interesting case for two reasons. If hypercharge is embedded in a simple grand unification group such as $SU(5)$, one would expect $k_1 = 5/3$. Even without a GUT embedding $k_1 = 5/3$ is interesting.

We recall that unification of gauge couplings is a necessary phenomenon in string theory. Specifically, at tree level, the gauge couplings of the different gauge groups are related to the string coupling constant $g_{\rm st}$ which is determined by the VEV of the dilaton field as [13]

$$k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = g_{\rm st}^2 \tag{7}$$

where k_i are the levels of the corresponding Kac–Moody algebra. In particular, if $k_1 : k_2 : k_3 = 5/3 : 1 : 1$, we would have $\sin^2\theta_W = 3/8$ at the string scale, a scenario identical to that of conventional gauge coupling unification with simple group such as $SU(5)$. For these reasons, we shall pay special attention to the case $k_1 = 5/3$.

An interesting example of a discrete gauge symmetry in the MSSM (or the SM) with seesaw neutrino masses is the Z_6 subgroup of $B-L$. The introduction of the right–handed neutrino for generating small neutrino masses makes $B-L$ a true gauge symmetry. When the ν^c fields acquire super-large Majorana masses, $U(1)_{B-L}$ breaks down to a discrete Z_6 subgroup. The Z_6 charges of the MSSM fields arising from $B-L$ are displayed in Table 1. Here we have used the standard notation for the fermion fields (Q and L being the left–handed quark and lepton doublets, u^c, d^c being the quark singlets, and e^c, ν^c being the (conjugates of) the right–handed electron and the right–handed neutrino singlets). To obtain the unbroken Z_6 charge, we first multiply te $B-L$ charge by 3 so that they become integers, then observe that the $\nu^c\nu^c$ Majorana mass term carries 6 units of this integer $B-L$ charge. Thus this Z_6 subgroup is left unbroken.

It is worth mentioning that the Z_6 symmetry has a Z_2 and a Z_3 subgroups as well. In the analysis that follows in the next section we will be making connections with the Z_6 subgroup of $B-L$ and its Z_2 and Z_3 subgroups.

Field	Q	u^c	d^c	L	e^c	ν^c	H_u	H_d
$\mathrm{U}(1)_{B-L}$	$1/3$	$-1/3$	$-1/3$	-1	1	1	0	0
Z_6	1	5	5	3	3	3	0	0

Table 1: The $B-L$ charges of the Standard Model fields along with the unbroken Z_6 subgroup after the neutrino seesaw.

Anomalous $U(1)$ symmetry has found applications in addressing the fermion mass and mixing hierarchy problem [14], doublet–triplet splitting problem in GUT [15], the strong CP problem [16], the μ problem of SUSY [17] and for SUSY breaking [18].

3 Discrete gauge symmetries in the MSSM

In this section we turn to the identification of discrete gauge symmetries in the MSSM that can serve as R–parity and simultaneously explain the origin of the μ term. We stay with the minimal particle content of MSSM, with the inclusion of the right–handed neutrinos needed for generating neutrino masses via the seesaw mechanism [19]. Anomalies associated with the discrete gauge symmetry will be cancelled by the Green–Schwarz mechanism as discussed in Sec. 2.

3.1 Constraints from the Lagrangian and discrete anomalies

We have displayed in Table 2 the particle content of MSSM along with their charges under an anomalous $U(1)$ gauge symmetry. The Grassmanian variable θ also carries a charge (equal to α), which allows for the $U(1)$ to be identified as an R symmetry. Z is a spurion superfield that acquires a non–zero F component and breaks supersymmetry with $\langle F_Z\rangle/M_{\rm Pl}\sim M_{\rm SUSY}\sim 10^2$ GeV. We shall assume family–independent $U(1)$ symmetry in this section. Any unbroken discrete symmetry must be family–independent to be consistent with MSSM phenomenology, that is the reason for focusing on such symmetries. In Sec. 4 we shall extend this analysis to flavor–dependent symmetries, even in that case, we will demand that a flavor–independent Z_M symmetry is left intact.

Field	Q	u^c	d^c	L	e^c	ν^c	H_u	H_d	θ	Z
SU(3)$_C$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
SU(2)$_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
U(1)$_Y$	1/6	−2/3	1/3	−1/2	1	0	1/2	−1/2	0	0
$U(1)_A$	q	u	d	l	e	n	h	$\bar{h}$	α	z

Table 2: The matter superfields of MSSM along with their anomalous $U(1)$ charges. θ is the Grassmann variable. Z is the spurion field which is responsible for supersymmetry breaking.

The superpotential of the model, including small neutrino masses via the seesaw mechanism is

$$W = Qu^cH_u + Qd^cH_d + Le^cH_d + L\nu^cH_u + M_R\nu^c\nu^c, \tag{8}$$

where M_R is the heavy right–handed neutrino Majorana mass. We have suppressed Yukawa couplings and generation indices, which must be understood.

In order to avoid a supersymmetric μ term in the Lagrangian, $\mathcal{L}\supset\mu\int d^2\theta H_uH_d$, so that the magnitude of μ may be related to the SUSY breaking scale, we impose the condition

$$h+\bar{h}\neq 2\alpha\ . \tag{9}$$

A μ–parameter of the right order is induced through the Guidice–Masiero mechanism [9] via the Lagrangian term, $\mathcal{L}\supset\int d^4\theta H_uH_d\frac{Z^*}{M_{pl}}$. Invariance of this term under the $U(1)$ symmetry requires

$$h+\bar{h}-z=0\ . \tag{10}$$

The gaugino masses arise through the Lagrangian term

$$\int d^2\theta W_\alpha W^\alpha\frac{Z}{M_{Pl}} \tag{11}$$

once $\langle F_Z\rangle\neq 0$ is induced. Combining the invariance of this term with that of the gauge kinetic term $\int d^2\theta W_\alpha W^\alpha$, we see that the spurion field Z must have zero charge under the

$U(1)$.[4] It is clear that the simultaneous presence of the gaugino mass term and the μ term reduces the $U(1)$ symmetry to a discrete subgroup Z_N. Therefore, one has to start with a discrete symmetry Z_N with the spurion superfield Z having a charge 0 mod N. Under Z_N, the conditions Eqs. (9)-(11) become

$$\begin{aligned} z &= 0 \;\; mod\ N \\ h+\bar{h} &= 0 \;\; mod\ N \\ h+\bar{h} &\neq 2\alpha \;\; mod\ N \end{aligned} \tag{12}$$

which also implies that $2\alpha \neq 0$ mod N.

Based on the invariance of the Yukawa couplings of Eq. (8) under Z_n and the conditions listed in Eq. (12), we obtain the following set of constraint equations:

$$\begin{aligned} z &= m_1 N \\ h+\bar{h} &= m_2 N \\ q+u+h &= 2\alpha + m_3 N \\ q+d+\bar{h} &= 2\alpha + m_4 N \\ l+e+\bar{h} &= 2\alpha + m_5 N \\ 2n &= 2\alpha + m_6 N \\ l+n+h &= 2\alpha + m_7 N, \end{aligned} \tag{13}$$

where m_i $(i = 1-7)$ are all integers.

The discrete Z_N anomaly coefficients for the $SU(3)_C$ and the $SU(2)_L$ gauge groups are

$$\begin{aligned} A_3 &= -3\alpha + 3q + \frac{3}{2}u + \frac{3}{2}d, \\ A_2 &= -5\alpha + \frac{9}{2}q + \frac{3}{2}l + \frac{1}{2}(h+\bar{h}) \ . \end{aligned} \tag{14}$$

Here we note that the fermionic charge of the u^c field, for example, relevant for the anomaly coefficient, is $(u-\alpha)$ since θ carries charge α. A_3 and A_2 include contributions from the gauginos as well. We shall cancel these anomalies by applying the Green–Schwarz mechanism as given in Eq. (6).

Non–zero gauginos masses arise through the VEV $\langle F_Z \rangle \neq 0$ (see Eq. (11)). Let us denote the Z_N charge of F_Z to be M. $\langle F_Z \rangle \neq 0$ breaks the original Z_N symmetry down to a subgroup Z_M:

$$Z_N \rightarrow Z_M \ . \tag{15}$$

(It must be that $M > 1$ for an unbroken discrete symmetry to survive after SUSY breaking.) Since $M = z - 2\alpha = m_1 N - 2\alpha$ where m_1 is an integer, we have

$$\alpha = \frac{n_1}{2} M \tag{16}$$

with n_1 being an integer. Let $N = n_0 M$ where n_0 is an integer. Since invariance of the Yukawa couplings under the Z_N symmetry requires invariance under the subgroup Z_M, we

[4] SUSY breaking scalar masses are invariant under the $U(1)$ symmetry and do not provide any constraint.

can solve the constraints of Eqs. (13)-(14) along with Eq. (6) to determine the various charges by first confining to the invariance under the smaller group Z_M. Under this Z_M, a superpotential term $\mu H_u H_d$ will be allowed. Once a solution is found, we can embed the Z_M symmetry into a higher Z_N symmetry that would forbid the μ term. Making some change of variables, viz., $n_2 = n_0 m_2$, $n_4 = n_0 m_6$, $n_5 = -n_0 p_1$, $n_6 = -n_0 p_2$, and applying the anomaly cancellation condition of Eq. (6), we obtain the charges of the various fields from Eqs. (13)-(14) as

$$\begin{aligned}
z &= M n_0 \\
h &= 3q + M\left(\frac{n_2 - n_6}{3} - \frac{n_4}{2} - \frac{7n_1}{6}\right) + \frac{M}{b}(n_2 - n_3 + \frac{n_5}{3} - n_1) \\
\bar{h} &= -3q + M\left(\frac{2n_2 + n_6}{3} + \frac{n_4}{2} + \frac{n_1}{6}\right) + \frac{M}{b}(-n_2 + n_3 - \frac{n_5}{3} + n_1) \\
u &= -4q + M\left(-\frac{n_2 - n_6}{3} + \frac{n_4}{2} + \frac{n_1}{6}\right) + \frac{M}{b}(-n_2 + n_3 - \frac{n_5}{3} + n_1) \\
d &= 2q + M\left(\frac{n_2 - n_6}{3} - \frac{n_4}{2} - \frac{7n_1}{6}\right) + \frac{M}{b}(n_2 - n_3 + \frac{n_5}{3} - n_1) \\
l &= -3q + M\left(-\frac{n_2 - n_6}{3} + \frac{2n_1}{3}\right) + \frac{M}{b}(-n_2 + n_3 - \frac{n_5}{3} + n_1) \\
e &= 6q + M\left(-\frac{n_2 - n_6}{3} - \frac{n_4}{2} - \frac{5n_1}{6}\right) + \frac{2M}{b}(n_2 - n_3 + \frac{n_5}{3} - n_1) \\
n &= M\left(\frac{n_4 + n_1}{2}\right) \\
\alpha &= M\frac{n_1}{2}\ .
\end{aligned} \tag{17}$$

Here we have defined $b \equiv k_3/k_2$. The n_i in Eq. (17) are all integers. A specific choice of the integers n_i will fix the charge assignment explicitly. We note that the terms proportional to q in Eq. (17) are proportional to the SM hypercharge Y. One can remove these terms and set $q = 0$ in Eq. (17) without loss of generality by making a shift of the Z_M charges proportional to Y. The quark doublet Q will then have zero charge under the unbroken Z_M. It should be kept in mind that to each solution we find, one can add Z_M charges proportional to Y to obtain equivalent solutions.

Based on Eq. (17), one can compute the anomaly coefficients under Z_M. They are

$$\begin{aligned}
A_3 &= \frac{3}{2} M(n_1 - n_2) \\
A_2 &= \frac{1}{2b} M(3n_1 - 3n_2 + b n_6)\ .
\end{aligned} \tag{18}$$

Note that from the last of Eq. (17), we have $2\alpha = 0\ mod\ M$. So the superpotential is invariant under Z_M. Also, under Z_M, one has $h + \bar{h} = 0\ mod\ M$, so a μ term in the superpotential is allowed by this symmetry. (Such a term will be forbidden when the Z_M symmetry is embedded into a higher Z_N symmetry, which we shall do in subsection 3.3.) In order to avoid R–parity breaking couplings, the total charges of the corresponding superpotential terms should be non–integer multiples of M, which puts extra constraints on the Z_M charges. There are four types of R–parity violating terms. Their Z_M charges are given by

$$\begin{aligned} u^c d^c d^c &: M\left(\frac{5n_1}{6} - \frac{2n_2+n_6}{3} - \frac{n_4}{2}\right) + \frac{M}{b}(n_2 - n_3 + \frac{n_5}{3} - n_1) \\ LLe^c &: M\left(\frac{n_1-n_4}{2}\right) \\ LH_u &: M\left(\frac{n_1-n_4}{2}\right) \\ QLd^c &: M\left(\frac{n_1-n_4}{2}\right). \end{aligned} \tag{19}$$

It is easy to show that the largest Z_M symmetry is Z_{6k_3} from Eq. (17). We shall now find solutions to these sets of equations for various values of the parameter $b = k_3/k_2$.

3.2 Green–Schwarz anomaly cancellation at Kac–Moody level 1

The simplest possibility for the parameters k_2 and k_3 to take is $k_3 = k_2 = 1$, so that $b = 1$ in Eqs. (17)-(18). This is the case of Kac–Moody algebra realized at level 1. Since the constraint equations depend only on the ratio $b = k_3/k_2$, the case of higher levels will coincide with that of level 1 as long as the levels are the same for both $SU(3)_C$ and $SU(2)_L$. From a theoretical point of view this case is the most attractive, since it allows for both $SU(3)_C$ and $SU(2)_L$ to emerge from the same gauge group as in a GUT. The charge assignment and possible discrete symmetries for this case $k_2 = k_3$ are shown explicitly in Table 3 [5].

The procedure we have followed to obtain Table 3 is as follows. First we set $b = 1$. Then we choose a set of integers n_i in the range $n_i \subset (0-5)$. Any n_i larger than or equal to 6 (or any negative n_i) can be absorbed into the *mod M* piece. The highest Z_M symmetry is then found to be Z_6. For every choice of the integer set n_i we demand that the R–parity breaking couplings of Eq. (19) be forbidden. (This requies $n_1 - n_4$ to be an odd integer and that the charge of d^c should be non–zero under Z_M.) Then we solve for the charges of the various fields, setting $q = 0$ as mentioned above. If the Green–Schwarz anomaly cancellation condition is satisfied we accept the solution. Upto overall conjugation and shifts proportional to hypercharge, the complete set of solutions is as ginven in Table 3. We have also listed the anomaly coefficients (A_2, A_3) in Table 3.

Several remarks are in order about the results shown in Table 3.

(i) Models I and II differ only in the value of α. In the effective low energy Lagrangian, what matters is 2α, which is the same for both models. Although the two models look

[5] Among the solutions, we remove those which either are conjugate of the listed solution or can be realized as linear combination of the known solution and hypercharge. For example, in model III and IV, we have chosen $q = 0 \;\; mod\ 6$. The charge q need not be actually zero. Since there exists an unbroken $U(1)_Y$ hypercharge which is anomaly–free, one can always take a linear combination of model III (or IV) with $U(1)_Y$ to find equivalent solutions with $q \neq 0 \;\; mod\ M$. This comment also applies to the models listed in all the other tables.

Model	Z_M	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
I	Z_2	2	1	1	2	1	1	1	1	1	(2, 2)
II	Z_2	2	1	1	2	1	1	1	1	0	(1, 1)
III	Z_6	6	5	1	2	5	3	1	5	3	(6, 6)
IV	Z_6	6	5	1	2	5	3	1	5	0	(3, 3)

Table 3: MSSM charge assignment when $k_2 = k_3$. α denotes the charge of the gaugino and the Grassmanian variable θ. When $\alpha \neq 0$, the Z_M acts as an R–symmetry. Also shown are the anomaly coefficients $(A_2,\ A_3)$.

identical from low energy point of view, their embedding into a high scale theory will not be the same. This is the reason for listing them separately. We shall see that when Z_M is embedded into a higher symmetry Z_N so as to forbid a large μ term, Models I and II will look different. Similar remarks apply to Models III and IV.

(ii) The Z_2 symmetries in Table 3 (I and II) are actually subgroups of the Z_6 symmetry (III and IV). Their embedding into Z_N will however lead to different solutions. Note also that the Z_3 subgroup of Z_6 does not show up as a solution, since that would allow for lepton number violation.

(iii) The Z_6 symmetric solutions of Table 3 are actually identical to the Z_6 subgroup of $B - L$ shown in Table 1 which can prevent R–parity violation in MSSM [20]. This can be recognized by taking linear combinations of Models III-IV and hypercharge. Suppose we normalize hypercharge so that all MSSM fields have integer values denoted by $\hat{Y}$ (with Q field having $\hat{Y} = 1$). Take now the combination $3(IV) + \hat{Y}$ $(mod\ 6)$. This redefined charge is identical to the Z_6 subgroup of $B - L$ of Table 1. The Z_2 models are identified as the Z_2 subgroups of $B - L$. We conclude from our systematic analysis that even with GS anomaly cancellation, the only allowed discrete symmetries at the Z_M level (which admits a superpotential μ term) are the subgroups of $B - L$. This will however be not the case when Z_M is embedded into Z_N. Note also that the anomaly coefficients A_2 and A_3 in Table 3 are all equal to $M/2$, so GS mechanism is not playing a role in anomaly cancellations. This remark will also be different in the Z_N embedding.

3.3 Embedding Z_M into Z_N and solving the μ–problem

We recall that the original Z_N symmetry broke down to a subgroup Z_M once the spurion field Z acquired a VEV along its F–component. At the level of Z_M, a superpotential μ–term is allowed. Now we turn to the task of identifying the original Z_N symmetry needed for explaining the μ term. We look for the simplest higher symmetry into which the Z_M solutions of Table 3 can be embedded. Each of the model in Table 3 has a different embedding into Z_N.

Consider the embedding of Model I in Table 3 into Z_N. The smallest Z_N group that contains a Z_2 subgroup is Z_4. This embedding is shown in Table 4. There are two possible charge assignments indicated as Models Ia and Ib. These models are obtained as follows. First we choose the value of α to be either 1 or 3 under Z_4 (since it must reduce to 1 under

Z_2). These two values correspond to the two models in Table 4. Then we demand that a bare μ term in the superpotential is prevented by the Z_4 symmetry. That determines the charges $(h,\ \bar{h})$ to be either (1, 3) or (3, 1). It turns out that the charges in the latter case are the conjugates of the former, and so we discard it. Then we set the charge n to be either 1 or 3, consistent with it being 1 under Z_2 subgroup. This fixes the charges of all fields. For each case the anomaly coefficients A_2 and A_3 are computed. If the anomalies are cancelled by the GS mechanism, we accept the solution. Only two solutions are found to survive, as displayed in Table 4.

Z_4	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
Ia	4	1	3	4	3	1	1	3	1	(1, 3)
Ib	4	1	3	4	3	1	1	3	3	(3, 1)

Table 4: Embedding of the Z_2 symmetry of Model I of Table 3 into Z_4 symmetry.

Note that the discrete Z_4 anomalies are cancelled by the GS mechanism. Individually A_2 and A_3 are not multiples of $N/2 = 2$, but the two coefficients differ only by $N/2 = 2$. We conclude that this simple solution would not have been possible without GS anomaly cancellation.

The models of Table 4 can be recast in a very simple form by forming the linear combination $\{\text{Ia} + \hat{Y}\ (mod\ 4)\}$, or $\{\text{Ib} + \hat{Y}\ (mod\ 4)\}$ with $\hat{Y}$ being the integer values of SM hypercharge. We display in Table 5 Model Ia recast in this form. The charge assignment is very simple, all matter fields of MSSM carry charge 1 under Z_4, while the Higgs superfields carry charge 0. The gauginos also have charge 1. The contribution to the Z_4 anomaly from the matter fields are the same for A_2 and A_3 (the number of color triplets is the same as the number of $SU(2)_L$ doublets in MSSM). While the gluino contributes an amount equal to 3 to A_3, the sum of the $SU(2)_L$ gaugino ($= 2$) and the Higgsinos ($= -1$) add to $A_2 = +1$. We see that A_2 and A_3 differ by $N/2 = 2$, signaling anomaly cancellation via GS mechanism.

Z_4	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
Ia	1	1	1	1	1	1	0	0	1	(3, 1)

Table 5: Model 1a recast with a shift proportional to hypercharge.

The charge assignment shown in Table 5 is clearly compatible with grand unification. The Kac–Moody level associated with hypercharge will be $k_1 = 5/3$ with a GUT embedding. Gauge coupling unification is then predicted, since $\sin^2\theta_W = 3/8$ near the string scale. This is true even if there were no covering GUT symmetry.

Now we turn to Model II of Table 3. Embedding this Z_2 into a Z_4 is not viable, since a large μ term cannot be prevented in that case. The next simplest possibility is Z_6, which also does not work as the Z_6 anomalies do not cancel. The simplest embedding is found to be into a Z_{10} with the charge assignment as shown in Table 6.

Z_{10}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
IIa	10	1	7	4	3	7	3	7	2	(6, 6)
IIb	10	7	9	8	1	9	1	9	4	(2, 2)

Table 6: Z_{10} embedding of Model II.

In the models of Table 6, one might consider a Z_5 subgroup of Z_{10}. This subgroup is sufficient to forbid the μ–term in the superpotential W, as well as to prevent dangerous R–parity violating couplings in W. With invariance only under Z_5, the term $u^c d^c d^c$ will have zero charge. A Lagrangian term arising from the Kahler potential $\mathcal{L} \supset \int d^4\theta (u^c d^c d^c Z^*/M_{\rm Pl}^2)$ will then be allowed. Once F_Z acquires a non–zero VEV, this term will lead to a superpotential term $\mathcal{L} \supset \int d^2\theta (M_{\rm SUSY}/M_{\rm Pl}) u^c d^c d^c$. Such a term violates R–parity, although very weakly. Signals of such a weak violation will be unobservable in collider experiments. However, this scenario will not fit well with cosmological constraints. The LSP will decay through this induced R–parity violating Yukawa coupling λ'' , which has a strength of order $M_{\rm SUSY}/M_{\rm Pl} \sim 10^{-15}$. We can estimate the lifetime of the LSP to be $\tau \sim [(\lambda'')^2 M_{LSP}/(8\pi)]^{-1} \sim 10^4$ sec. Such a lifetime falls into the cosmologically disfavored range and would be in violation of nucleosynthesis constraints. (This situation is analogous to the gravitino problem of supergravity, but is slightly worse, since the LSP mass is expected to be order 100 GeV, rather than a TeV for the gravitino, making the LSP lifetime somewhat longer than that of the gravitino.) We consider the Z_5 solution to be unacceptable for this reason. Since Z_5 symmetry does not contain a Z_2 subgroup, exact R–parity could not be defined after SUSY breaking, unlike in the case of Z_{10} model.

In Tables 7 and 8 we show the simplest embedding of Models III and IV into Z_{12} and Z_{18} respectively. The procedure we have adopted is identical to that for Models I and II.

Z_{12}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
IIIa	12	5	7	8	11	9	1	11	3	(9, 9)
IIIb	12	11	1	8	5	3	7	5	3	(9, 9)
IIIc	12	5	7	8	11	9	1	11	9	(3, 3)
IIId	12	11	1	8	5	3	7	5	9	(3, 3)

Table 7: Z_{12} embedding of Model III.

As in the case of the Z_{10} model of Table 6, we may consider taking a Z_9 subgroup of Z_{18} in Table 8. However, since Z_9 does not contain Z_2 or Z_6 as subgroups, after SUSY breaking, small R–parity violating Yukawa couplings of the type $W \supset LLe^c$ will be generated from the Kahler potential with coupling constants of order 10^{-15}. Such couplings would violate constraints from big bang nucleosynthesis since the lifetime of the LSP will be of order 10^4 sec. We shall not consider the Z_9 subgroup any further.

Z_{18}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
IVa	18	11	13	14	17	15	1	17	6	(9, 9)
IVb	18	5	1	8	11	15	7	11	6	(18, 9)

Table 8: Z_{18} embedding of model IV.

3.4 Discrete anomaly cancellation at higher Kac–Moody level

Thus far we have assumed the parameter $b \equiv k_3/k_2 = 1$. This is the case when the Kac–Moody levels for $SU(3)_C$ and $SU(2)_L$ are the same, the simplest possibility being $k_3 = k_2 = 1$. It is also possible that k_2 and k_3 are not the same. It is not clear to us how easy it is to construct string models with different values for k_3 and k_2. Although it might appear less attractive theoretically, it is nevertheless a logical possibility. In this section we analyze discrete anomaly cancellation for values of $k_3/k_2 \neq 1$.

From a technical point of view it appears to be difficult to construct models with levels higher than 3 in string theory [21]. Motivated by this observation, we shall confine our discussions to k_2 and k_3 being less than or equal to 3. This allows for the cases when $b \equiv k_3/k_2 = 1, 2, 1/2, 1/3, 2/3$ and $3/2$. The case of $b = 1$ has already been analyzed in the previous section, so we turn to the other cases.

From the solution Eq. (17) which applies to Z_M invariance (that allows a bare μ term, but forbids all R–parity violations), a few simplifications can be found. The case where $b = 1/2$ and $b = 1/3$ are identical to the case of $b = 1$. This is because the b–dependent terms only contribute to the various charges proportional to n_5 in Eq. (17). But this n_5 contribution can be absorbed into the n_2 term in all equations. No new solutions will then be generated under Z_M. Similarly, it is easy to see that the cases $b = 2/1$ and $b = 2/3$ are equivalent under Z_M. And the case where $b = 3/1$ becomes identical to the case of $b = 3/2$. Among these equivalent cases under Z_M, we shall only consider one possibility. Although it is possible that when the resulting models are embedded into a higher symmetry Z_N, new models at higher levels may emerge, we shall not pursue it here.

We shall then focus on the case where $b \equiv k_3/k_2 = 2$, and $b = 3$ for anomaly cancellation at higher Kac–Moody level. Following the same procedure as in the previous section, we obtain the corresponding discrete symmetry and charge assignment. The solutions for the case of $k_3/k_2 = 2$, which is the same for $k_3/k_2 = 2/3$, are shown in Table 9. The discrete Z_M symmetry is Z_6 in this case. Note that the discrete GS mechanism cancels the gauge anomalies of Z_6. For example, $A_2 = 9/2,\ A_3 = 6$ is anomaly free since with $k_2 = 1,\ k_3 = 2$, the cancellation condition is that $2A_2$ and A_3 differ by an integer multiple of $N/2 = 3$ (see Eq. (6)).

The two Z_6 models of Table 9 have been embedded into the simplest possible Z_N model in Tables 10 and 11. The Z_N symmetries are found to be Z_{12} and Z_{18}. The discrete gauge anomalies are cancelled by GS mechanism, as before. Take Model Vd for example, which has $A_2 = 3/2,\ A_3 = 9$ under Z_{12}. $2A_2$ and A_3 differ by 6, which is an integer multiple of $N/2 = 6$.

The next case is when $b = k_3/k_2 = 3$, which gives at the Z_M level the same models

Model	Z_M	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
V	Z_6	6	2	4	5	5	3	4	2	3	(9/2, 6)
VI	Z_6	6	2	4	5	5	3	4	2	0	(3/2, 3)

Table 9: Discrete symmetries and the corresponding charge assignment when $k_3/k_2 = 2$ or $2/3$.

Z_{12}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
Va	12	8	4	5	11	3	10	2	3	(9/2, 9)
Vb	12	2	10	5	5	9	4	8	3	(9/2, 9)
Vc	12	2	10	11	11	3	4	8	3	(3/2, 9)
Vd	12	8	4	11	5	9	10	2	3	(3/2, 9)

Table 10: Embedding of Z_6 of Model V into Z_{12}.

Z_{18}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
VIa	18	2	10	11	5	15	4	14	6	(9/2, 18)
VIb	18	14	16	5	17	15	10	8	6	(27/2, 9)

Table 11: Z_{18}–embedding of the Z_6 model VI.

as $b = 3/2$. In Table 12 we list the allowed Z_M models, with $M = 18$. Table 13 has the embedding of Model VII into Z_{18} that prevents a large μ term, Table 14 has the embedding of Model VIII, the simplest possibility for which being Z_{90}. In all cases the discrete anomalies are cancelled by the GS mechanism.

Model	Z_M	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
VII	Z_{18}	18	1	17	10	7	9	17	1	9	(6,18)
VIII	Z_{18}	18	1	17	10	7	9	17	1	18	(15, 9)

Table 12: Discrete symmetries and the charge assignments for $k_3/k_2 = 3$ or $3/2$.

Z_{36}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
VIIa	36	1	35	28	25	9	17	19	9	(33, 27)
VIIb	36	19	17	10	7	9	35	1	9	(6, 27)

Table 13: Z_{36}–embedding of the Z_{18} model VII.

Z_{90}	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
VIIIa	90	55	71	46	25	9	53	37	54	(89, 27)
VIIIb	90	55	53	28	25	27	89	1	72	(42, 36)

Table 14: Embedding of Z_{18} of Model VIII into Z_{90}.

It should be mentioned that at the level of Z_M, it is easy to realize an R–parity that allows for lepton number violation, but conserves baryon number. Rapid proton decay will be prevented in this case. Lepton number violating processes and neutrino masses do provide some constraints, but these are much less stringent.

Consider the Z_6 models of Table 3 (Models III and IV). Suppose we impose invariance only under the Z_3 subgroup of Z_6 with the same charge assignment as in Table 3. The lepton number violating couplings $LH_u,\ LLe^c$, and QLd^c all have charge 3 under the Z_6 symmetry, so with only Z_3 invariance imposed, these couplings will be allowed in the superpotential. Since the original Z_6 symmetry is free from discrete gauge anomalies, the subgroup Z_3 is also free from such anomalies. One cannot however embed this Z_3 symmetry to any higher Z_N in order to explain the μ–parameter. Consider the LH_u term in the superpotential. Z_N invariance of this term would imply $l + h = 2\alpha \ mod\ N$. The last two relations of Eq. (13) would imply $2\alpha = 0 \ mod\ N$, implying that a bare μ term in the superpotential will be allowed. An alternative explanation for the μ–term will have to be found in the case of lepton number violating R–parity.

It is also possible, although somewhat non–trivial, to have baryon number violating R–parity without dangerous lepton number violation. (Neutrino masses violate lepton number by two units, but that does not result in rapid proton decay.) At the level of Z_N we can show that anomalies associated with such an R–parity will have to be cancelled at higher Kac–Moody levels. If the coupling $u^c d^c d^c$ is allowed in the superpotential, we find that the Z_N discrete symmetry has anomalies given by $A_3 = 3\alpha$, $A_2 = 5/2\alpha - (3/4)pN$ where p is an integer. Imposing the anomaly cancellation condition, Eq. (6), we find $\alpha = (m_1 k_2 + m_2 - (k_3/2)p)N/(6k_3 - 5k_3)$ with m_1, m_2, p being integers. When $k_2 = k_3 = 1$, this relation shows that $2\alpha = 0 \; mod \; N$, meaning that a bare μ–term will be allowed in the superpotential. If we choose k_2 and k_3 differently, this problem will not arise. Consider for example, $k_3 = 2, k_2 = 1$. A consistent Z_N charge assignment corresponding to a Z_4 symmetry is shown in Table 15 for this case. This model allows for the coupling $u^c d^c d^c$, while preventing other R–parity violating couplings. The Z_4 anomalies are cancelled by GS mechanism, which in this case reads as $2A_2 - A_3 = m - 2n$, with m, n being integers.

Z_4	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
A	4	2	2	1	1	1	2	2	1	(1/2, 3)
B	0	2	2	3	3	3	0	0	3	(5/2, 5)

Table 15: Examples of a Z_4 symmetry that allows for baryon number violation without dangerous lepton number violations. The Z_4 anomalies are cancelled by GS mechanism at levels $k_2 = 1, k_3 = 2$.

4 Discrete flavor symmetries and the the fermion mass hierarchy

As indicated earlier, there must be an unbroken Z_M symmetry which is flavor–independent that survives to low energy scales, to be identified as an R–parity. We can however introduce flavor dependence in the original symmetry, provided that a subgroup of the flavor group remains unbroken and can be identified as one of the Z_M symmetries of Table 3. In this section we embark on this question. Our aim will be to seek an understanding of the observed hierarchy in the fermion masses and mixings without introducing such hierarchy by hand.

Anomalous $U(1)_A$ symmetry is widely used for the explanation of fermion mass and mixing hierarchy [22]. The general superpotential in this has the following expression:

$$\begin{aligned} W &= Q_i u_j^c H_u \left(\frac{S}{M_{\rm Pl}}\right)^{(h_1)_{ij}} + Q_i d_j^c H_d \left(\frac{S}{M_{\rm Pl}}\right)^{(h_2)_{ij}} + L_i e_j^c H_d \left(\frac{S}{M_{\rm Pl}}\right)^{(h_3)_{ij}} \\ &+ L_i \nu_j^c H_u \left(\frac{S}{M_{\rm Pl}}\right)^{(h_4)_{ij}} + \nu^c \nu^c S \left(\frac{S}{M_{\rm Pl}}\right)^{(h_5)_{ij}} \end{aligned} \tag{20}$$

where S is an MSSM singlet field with a non–trivial anomalous $U(1)_A$ charge. S acquires a VEV near the string scale and disappears from the low energy spectrum. Here $(h_\alpha)_{ij}$ is a set of integers for $\alpha = 1, 2, 3, 4, 5$ and $i, j = 1, 2, 3$ are the generation indices. We assume that all the Yukawa couplings are of order one. After S field develops a VEV, near but somewhat below the string scale, a small parameter $\epsilon = \langle S \rangle / M_{\rm Pl} \sim 1/5$ is generated. This factor appears in various powers with the Yukawa couplings, explaining the observed mass and mixing hierarchy [22]. It is possible to suppress all the MSSM Yukawa couplings to the desired level by choosing appropriate set of $U(1)$ charges [14].

An acceptable flavor texture which gives the correct pattern of fermion masses and mixings is:

$$\begin{aligned} U_{ij} &= \begin{pmatrix} \epsilon^6 & \epsilon^5 & \epsilon^3 \\ \epsilon^5 & \epsilon^4 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} H_u, \qquad D_{ij} = \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^p H_d, \\ L_{ij} &= \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon \\ \epsilon^3 & \epsilon^2 & 1 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \epsilon^p H_d, \qquad \nu^D_{ij} = \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_1} H_d, \end{aligned} \tag{21}$$

where U_{ij}, D_{ij}, L_{ij} and ν^D_{ij} correspond to up–quark, down quark, charged lepton and Dirac neutrino Yukawa matrices resulting from the appropriate powers of the S field in Eq. (20). The integer p can be either 0, 1 or 2, corresponding to large, medium and small $\tan\beta$ respectively.

Once the charged lepton sector and Dirac neutrino sector are constructed, we can uniquely define the form of the heavy Majorana neutrino mass matrix. In the present example it is

$$\nu^M_{ij} = \begin{pmatrix} \epsilon^2 & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix} \epsilon^{a_2} \ . \tag{22}$$

As mentioned before, the MSSM superpotential does not possess any unbroken $U(1)$ symmetry, apart from the gauge symmetry. Therefore we seek solutions of a Z_N discrete symmetry that would generate the Yukawa matrices of Eq. (21).

Z_N invariance of the Yukawa couplings in Eq. (20) imposes constaints in the up–quark sector given by

$$q_i + u_j + h + ps = 2\alpha \ mod\ N, \tag{23}$$

where p is the power of ϵ appearing in the appropriate element of the U_{ij} matrix, which is equal to the power in the field S. s denotes the $U(1)$ charge of S field. Similar conditions apply for the charges of the other SM fermions as well. By construction, the flavor structure of the matrices obey the "determinant rule", viz., that in any 2×2 sub–block, the determinant is a homogeneous function of ϵ. This means that out of the 18 conditions for the up–quark and the down–quark sector, only 8 will be independent.

We wish to have an unbroken Z_M symmetry that is a subgroup of Z_N which is flavor–independent. Since the flavon field S has a Z_N charge of s, once it acquires a VEV of order the string scale, the Z_N will be broken down to Z_s. We shall attempt to embed the Z_M of Table 3 into Z_s.

To be specific, let us work out an example with the Z_2 model of Table 3 and embed this Z_2 into a higher Z_N symmetry that allows for the desired flavor structure. The Z_N symmetry

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d	θ	S	A_2	A_3
A	0,2,6	1,3,7	3,5,5	4,6,6	13,1,5	5,7,7	1	13	7	2	6	13
B	4,6,10	13,1,5	11,13,13	6,8,8	9,11,1	5,7,7	13	1	7	2	13	13
C	6,8,12	5,7,11	1,3,3	0,2,2	7,9,13	5,7,7	9	5	7	2	13	6

Table 16: Examples of flavor–dependent Z_{14} symmetry which forbids all $R-$parity breaking terms. $i = 1,2,3$ is the flavor index and charges in the brackets are in order of 1-3. We are considering $p = 2$ and $q = 0$ in Eq. (21) which corresponds to medium values of $\tan\beta \sim 10$. We have taken $a_2 = 0$ in Eq. (22) for simplicity.

must be Z_{14+2n} for this embedding to be consistent, with n being any integer. The smallest such symmetry is then Z_{14}. The flavon field S must carry zero charge under Z_2 and should transform non–trivially under the Z_N. The simplest possibility is $s = 2$. Now, the Yukawa textures of Eq. (21) makes use of S^6 terms in the superpotential, which should be different from S^0. This requirement makes the smallest Z_N symmetry to be Z_{14}. If this symmetry were Z_{12}, for example, S^6 will be neutral under Z_{12}, making the (11) and the (33) entries of the up–quark Yukawa matrix of the same order. We can generalize this statement to any low scale discrete symmetry. The corresponding flavor–dependent symmetry must be $Z_{M(k+1)+n}$, where Z_M is the low scale surviving discrete symmetry, k corresponds the highest power of the S field in the general superpotential, Eq. (20), and n is a positive integer. This choice will guarantee the existence of Z_M discrete R–parity at low energy scales.

Three examples of Z_{14} symmetric models are presented in Table 16. We have chosen the charge of S to be 2 and fixed the charge of θ to be 7 in these examples. Discrete anomaly cancellation is enforced via GS mechanism at Kac–Moody level 1. We have also imposed the conditions that the Z_{14} symmetry forbid all R–parity violating couplings.

The Z_N symmetry group would depend on the highest power of ϵ appearing in the fermion Yukawa matrices. If we want to have symmetry smaller than Z_{14}, we should reduce the power of S field in Eq. (20). One way is to re-parameterize the value of ϵ. For example, if ϵ is taken to be of order 1/10, rather than 1/5 as was assumed in Eq. (21), it might suffice to use cubic powers of S at most. A Z_8 discrete symmetry would then suffice to forbid the R–parity breaking terms. We consider the expansion given in Eq. (21) to be more realistic.

In Table 17 we present three models based on Z_{28} symmetry that forbid all R–parity violating couplings, explain the fermion mass and mixing hierarchy via the texture of Eq. (21) and also solve the μ–problem via the Guidice–Masiero mechanism. As before, the discrete gauge anomalies are cancelled by the GS mechanism. We find it remarkable that a single discrete symmetry can do all these jobs. It may be mentioned that Z_{28} is not a large symmetry unlikely to be realized in string theory. For example, if the particle spectrum contains fields carrying charges of $(1, 1/4, 1/7)$ under the anomalous $U(1)$, and if a scalar field with charge 1 acquires a VEV, the unbroken Z_N symmetry will be Z_{28}.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	H_u	H_d	θ	S	A_2	A_3
A	12,16,24	7,11,19	9,13,13	4,8,8	17,21,1	3,7,7	27	1	7	4	11	11
B	22,26,6	23,27,7	1,5,5	2,6,6	21,25,5	3,7,7	1	27	7	4	11	11
C	26,2,10	7,11,19	9,13,13	18,22,22	17,21,1	3,7,7	13	15	7	4	11	25

Table 17: Examples of flavor–depended discrete Z_{28} symmetry which prevent R–parity breaking couplings, explain the origin of the μ–term via the Guidice–Masiero mechanism, and explain the hierarchy in quark and lepton masses and mixings via the Yukawa texture shown in Eq. (21). $i = 1, 2, 3$ is the flavor index and charges in the brackets are in order of 1-3. We are considering $p = 2$ and $q = 0$ in Eq. (21), which corresponds to medium values of $\tan\beta \sim 10$. We have taken $a_2 = 0$ in Eq. (22) for simplicity.

5 Conclusion

In this paper we have investigated the possibility of realizing R–parity of MSSM as a discrete gauge symmetry. Simultaneously we have demanded that this discrete symmetry should provide a natural explanation for the μ–term, the Higgsino mass parameter in the MSSM superpotential, via the Guidice–Masiero mechanism. We have adopted a discrete version of the Green–Schwarz anomaly cancellation mechanism in our search for discrete gauge symmetries, which is is less constraining than the conventional methods.

We have found simple examples of Z_N symmetries that act as R–parity and simultaneously solve the μ–problem, without extending the particle content of the MSSM. The simplest example is a Z_4 symmetry with a simple charge assignment that is compatible with grand unification. The Green–Schwarz mechanism plays a crucial role in cancelling the Z_4 anomalies. Other examples of Z_N symmetries are provided with $N = 10, 12, 18, 36$ etc. In some cases the discrete anomalies are cancelled by the GS mechanism at higher Kac–Moody levels. We have found that it is easy to realize lepton number violating R–parity as a discrete symmetry, but implementing the Guidice–Masiero mechanism for the μ term is difficult in this case. Baryon number violating R–parity can be realized, along with a natural explanation of the μ term, but the discrete gauge anomalies are cancelled in this case at higher Kac–Moody level.

It has been shown that a simple Z_N symmetry can also explain the observed hierarchical structure of quark and lepton masses and mixings, while preserving the desired R–parity and the solution to the μ–problem. Examples of a Z_{28} symmetry doing all these have been presented.

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